The UNNS Quantization Protocol (UQP): Recursive Operators as Quantum Fields

UNNS Research Notes

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Abstract

We introduce the *UNNS Quantization Protocol* (UQP), which extends the Hamiltonian Protocol (UHP) by quantizing recursion. Nests are elevated to states in a Hilbert space, operators act as creation and annihilation operators, and the zero nest plays the role of vacuum. This protocol develops a recursive analogue of quantum field theory, providing spectra, excitations, and ground states within UNNS.

Contents

1 Motivation

The Hamiltonian Protocol provided energy and phase space. Quantization requires operator algebras, ground states, and spectra. The UQP equips UNNS with quantum-like structure, allowing recursive phenomena to be studied as excitations above a substrate vacuum.

2 Hilbert Space of Nests

Definition 2.1 (Recursive Hilbert Space)**.** *Let* $\mathbb{H}_{\text{UNNS}}$ *be the Hilbert space spanned by basis vectors* $\{|\mathcal{N}\rangle\}$ *for admissible nests* $\mathcal{N}$*, with inner product*

$$\langle \mathcal{N}_1 | \mathcal{N}_2 \rangle = \delta_{\mathcal{N}_1, \mathcal{N}_2}.$$

Remark 2.2. *This defines a discrete orthonormal basis of nest states.*

3 Vacuum and Operators

Definition 3.1 (Vacuum State)**.** *The zero nest defines the vacuum:*

$$|0\rangle \equiv |\mathcal{N} = 0\rangle.$$

Definition 3.2 (Creation and Annihilation Operators)**.** *For each coefficient a_i of a nest:*

$$\hat{a}_i^\dagger |\mathcal{N}\rangle = |\mathcal{N} + e_i\rangle, \qquad \hat{a}_i |\mathcal{N}\rangle = \langle a_i \rangle \, |\mathcal{N} - e_i\rangle,$$

where e_i denotes the unit increment at depth i.

Proposition 3.3 (Commutation Relations)**.** *The recursive operators satisfy*

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}, \qquad [\hat{a}_i, \hat{a}_j] = [\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0.$$

Remark 3.4. *This makes $\mathbb{H}_{\text{UNNS}}$ a Fock space of recursive states.*

4 Recursive Hamiltonian Operator

Definition 4.1. *The Hamiltonian operator is*

$$\hat{H} = \sum_i \omega_i \, \hat{a}_i^\dagger \hat{a}_i,$$

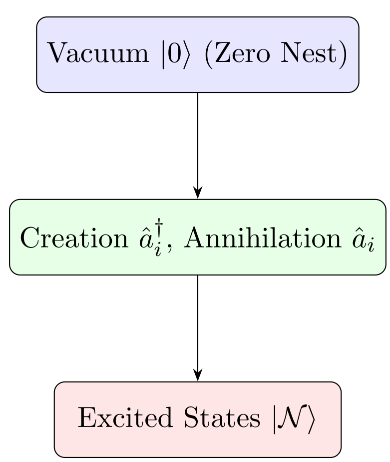
where ω_i encodes the recursion frequency at depth i.

Theorem 4.2 (Energy Spectrum)**.** *The spectrum of $\hat{H}$ is*

$$E = \sum_i \omega_i n_i,$$

where n_i counts excitations at coefficient a_i.

5 Diagrammatic Overview



6 Applications

6.1 Mathematics

- Defines a recursive Fock space.
- Provides operator algebra for nests.

6.2 Physics

- Parallels quantum harmonic oscillator with recursion excitations.
- Suggests UNNS quantum field analogs.

6.3 Computation

- Models recursion states as quantum states.
- Applications in quantum algorithms for recursive systems.

7 Conclusion

The UNNS Quantization Protocol establishes a quantum analogue for recursion, defining vacuum, creation, annihilation, and Hamiltonian spectra. It aligns UNNS with quantum field theory concepts, offering new vistas in mathematics, physics, and computation.